III. The STM-8 Loop - *Work in progress*

This contains the loop functions for a velocity input. The interaction between, forward gain, feedback function, loop gain and transfer function are well displayed. The asymptote functions give a simple way to estimate loop performance. More detail to mathematics. Some general design considerations added.

This is an analysis of the feedback loop of the STM-8 vertical seismometer. The results agree closely with the MathCad analysis done by S-T Morrissey. My goal here is to clarify, mostly in my own mind, the structure of the feedback system, and to that end I want to block-diagram the system in the form that I am used to using. In particular, I would like to derive a view of the loop which is aimed as much toward the synthesis process as it is toward analysis. For more information on feedback analysis, see the feedback tutorial file “feedback.pdf”.

The analysis is done with respect to an inertial frame of reference fixed to the earth’s surface. The input signal is the force on the mass due to ground motion. The feedback is the force from the Feedback Coil. The mass itself is the summing point.

There are three branches in the feedback path. The two outputs on the right, in blue, are independent of the loop and can be analyzed separately; that is, they are independent if there are no significant loading effects, which I think is the case.

The next step will be to insert the appropriate transfer functions for each block. The spring Mass will be a quadratic, the others will be quite simple functions.

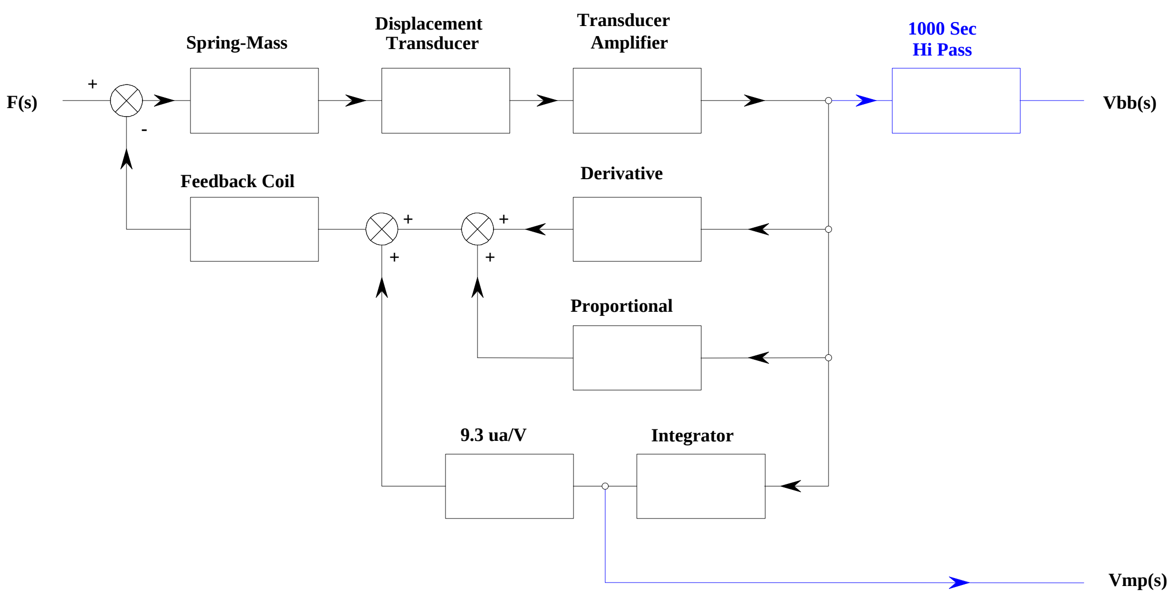

Figure 2 – Original STM-8 configuration

To analyze this loop we want to reduce it to the form of the Basic Loop, in Figure 1 (in "feedback.pdf") . Note that any two blocks in the diagram in which an output connects to a single input, can be combined, by multiplying their transfer functions together and assigning the result to the combined block.

First we need to deal with the Mass Position output. We want to detach it from the feedback loop. To do that we just have to make a copy of the integrator block.

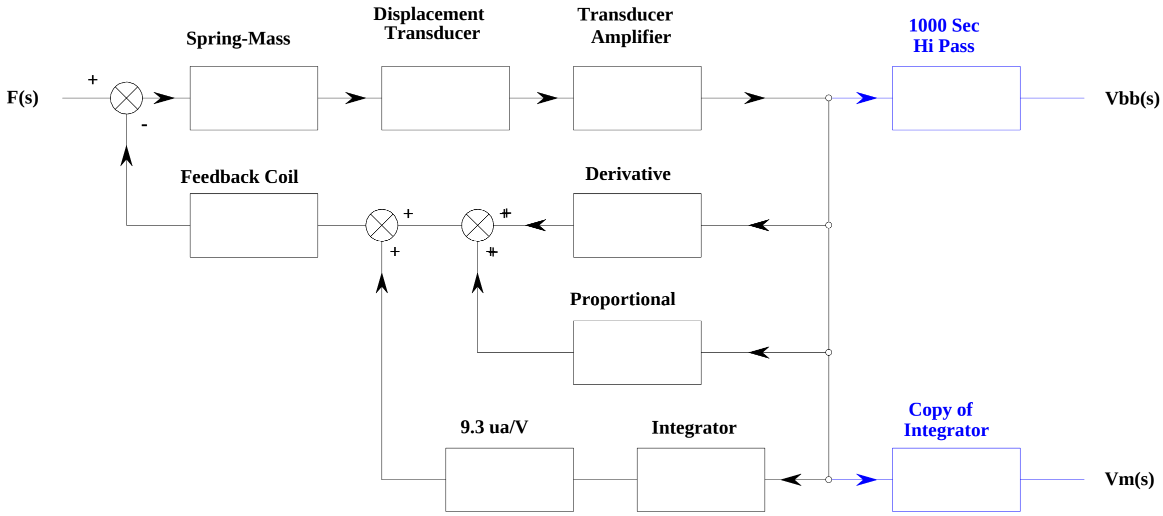

Figure 3 – Mass Position integrator duplicated

Finally because the coil resistance is low compared with the three feedback branches, they can be easily combined into a single block with its transfer function being the sum of the transfer functions of the three branches.

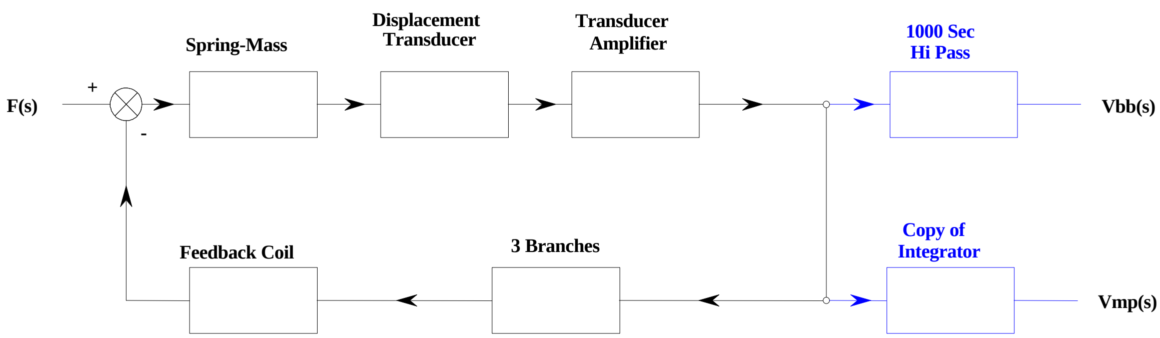

Figure 4 – Feedback branches combined

Transfer functions

In the following, the symbol ω is used to represent frequency in radians per second.
The symbol f represents frequency in Hz, where $\omega = 2\pi f$
Similarly, $\tau \equiv 1/\omega$ is the time constant sec/radian and T is the period in seconds. $T = 2\pi\tau$
The symbol s is the complex variable somewhat related to frequency

Spring-mass

$$\frac{x(s)}{Force(s)} = \frac{1}{K\,(s^2\,M/K + s\,R/K + 1)} \quad \text{meters / Newton}$$

where x(s) = mass position
Force(s) = external force applied to mass - Newtons
R is a velocity damping term Newtons / meter/sec
K = effective spring constant at mass M location = dF/dx - Newtons/meter

With other variables

$$\frac{x(s)}{Force(s)} = \frac{1}{K\,(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{m / N}$$

where τ_0 is the time constant. $\tau_0 = \sqrt{M/K}$ - sec/*rad* *May be used to compute* K *from* T_0
where T_0 is the undamped natural period in seconds
and ζ is the damping factor. $\zeta = \frac{R}{2\sqrt{MK}}$ *May be used to compute the value of* R

In terms of acceleration, using F = MA

$$\frac{x(s)}{Acc(s)} = \frac{M}{K\,(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{m / m/s}^2$$

where M = effective seismic mass - Kg
and Acc(s) = acceleration of the mass - m/s^2

In terms of velocity
Acc(s) = dVel(s)/dT = s Vel(s) *Multiplying by s corresponds to differentiation*

$$\frac{x(s)}{Vel(s)} = \frac{M\,s}{K\,(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{m / m/s}$$

where Vel(s) = velocity of the mass - m/s

Displacement Transducer
3.7648E+4 Volts/meter *DC to several hundred hz. Look for phase shifts at high end.*

Transducer Amplifier
x 10 + input filter terms *Filters not included. Effect mostly at f > 100 Hz*
r = Total displacement transducer constant = 10 x 3.7648E4 $\cong$ 3.7E5 Volts/Meter

Total Forward Path expression - *Spring-Mass and displacement transducer*

$$A(s) = \frac{r}{K\,(s^2 \tau_0^2 + s\, 2\zeta\tau_0 + 1)} \quad \text{Volts / N}$$

where τ_0 is the time constant. $\tau_0 = \sqrt{M/K}$ - sec/*rad*

where T_0 is the undamped natural period in seconds

and ζ is the damping factor. $\zeta = \frac{R}{2\sqrt{MK}}$.

Expressing A(s) in terms of velocity,

$$A(s) = \frac{s\,M\,r}{K\,(s^2 \tau_0^2 + s\, 2\zeta\tau_0 + 1)} \quad \text{Volts/ m/s}$$

Integrator – *Assumes feedback coil resistance is low vs. 107k* Ω

$$\frac{I_{out}(s)}{V_{in}(s)} \cong \frac{(1/\tau_I)}{R_I\,(s + 1/\tau_I)} \quad \text{Amperes / Volt sec}$$

$R_I = 1.07E5$, Ra = 2E6, Ca = 40.2E-6

where τ_I = RaCa = 80.4 sec/*rad*

or substituting $\omega_I = 1/\tau_I$

$$\frac{I_{out}(s)}{V_{in}(s)} \cong \frac{\omega_I}{R_I\,(s + \omega_I)} \quad \text{Amperes / Volt sec}$$

Proportional Feedback - *Assumes feedback coil resistance is low vs. 581k* Ω

$$\frac{I_{out}(s)}{V_{in}(s)} \cong 1 / R_p \qquad R_p = 5.81E5 \text{ ohms}$$

$$= 1.72E\text{-}6 \quad \text{Amperes / Volt}$$

Derivative Feedback

$$\frac{I_{out}(s)}{V_{in}(s)} \cong sC_d \quad \text{Amperes/ Volt/sec or} = \frac{sC_d}{(sR_cC_d + 1)} \quad \text{including coil-resistance effects}$$

C_d = 24.1 μf, R_c = Feedback coil resistance = 8 Ohms

Feedback coil

$$G_n \equiv \frac{Force(s)}{I_{in}(s)} = 12.98 \text{ Newtons /Ampere}$$ *Assumed wide-band.*

Total Combined Feedback expression - *Assuming feedback coil resistance is low.*

$$B \equiv \frac{Force(s)}{V_{in}(s)} \cong Gn\left(sC_d + \frac{1}{R_p} + \frac{1}{R_I}\frac{\omega_I}{(s + \omega_I)}\right) \quad \text{Newtons / Volt}$$

Expressed in terms of velocity

$$B \equiv \frac{Vel(s)}{V_{in}(s)} \cong \frac{Gn}{M}\,\frac{1}{s}\left(sC_d + \frac{1}{R_p} + \frac{1}{R_I}\,\frac{\omega_I}{(s+\omega_I)}\right) \quad \text{meters/sec / Volt}$$

Expressed in terms of time constant τ_B and damping factor ζ_B

$$B \cong \frac{G_n}{M}\,\frac{(R_I + R_p)}{R_I\,R_p}\,\frac{1}{s}\,\frac{\omega_I}{(s+\omega_I)}\left(s^2\,\tau_B^{\,2} + s\,2\zeta_B\tau_B + 1\right) \quad \text{meters/sec / Volt}$$

where $\tau_B^{\,2} = \frac{C_d\,R_I}{\omega_I}\,\frac{R_p}{(R_p + R_I)}$ and $\zeta_B = \frac{1}{2}\tau_B\left(\omega_I + \frac{1}{R_p\,C_d}\right)$

Note that with the proportional feedback removed $(Rp \rightarrow \infty)$ these reduce to

$$\tau_B^{\,2} = \frac{C_d\,R_I}{\omega_I} \quad \text{and} \quad \zeta_B = \frac{1}{2}\,\tau_B\,\omega_I$$

Mass position filter – 1000 sec hi-pass

$$\frac{s}{(s + 1/\tau_P)} = \frac{s}{(s+\omega_p)} \qquad R_p = 1E6, \qquad C_p = 1000E\text{-}6$$

where $\tau_P = R_pC_p = 1000$ sec/*rad*. $\omega_p \equiv 1/\tau_p = 0.001$ *rad*/sec

VBB output amplifier.

$$\frac{(s + (1/\tau_1 + 1/\tau_2)}{(s + 1/\tau_2)}$$

where $\tau_1 = R_1C_1$, $\tau_2 = R_2C_1$, $R_1 = 4.7E5$, $R_2 = 2E6$, $C_1 = 1E\text{-}7$
so $\tau_1 = 4.7E\text{-}2$ sec/*rad*.; $\tau_2 = 0.2$ sec/*rad*.

Closed Loop Transfer Function: = System response with respect to applied input velocity
CLTF = A(s)/(1 + A(s)B(s))

For those frequencies where loop gain AB is high, the closed loop transfer function $V_{out}(s) / Vel_{in}(s) \cong 1/B(s)$ Volts / meter/sec, where $V_{out}(s)$ is the output signal voltage and $Vel_{in}(s)$ the the input velocity due to ground motion. The system functions for a velocity input to the STM-8 are graphed in figure 5.

The graphs include A, the forward portion of the loop, made up of the spring-mass system, the displacement transducer and its amplifier. This plot currently ignores the effect of the filters following the synchronous detector, but its effect is rather small, and is in the region above 100 hz. The spring-mass was arbitrarily given a damping factor of 0.1, with an undamped natural frequency of 0.5 Hz. That frequency with a mass of 0.5kg implies an effective spring-constant K = 4.935 N/m. The damping factor determines the sharpness and height of the peak in the A function but will not show up in the CLTF.

The system response to a velocity input, the CLTF of Figure 5, is very similar to the one given by S-T Morrissey in his analysis. In the mid-frequencies the gain is computed here as 1590 V / m/s.

Gain-crossover is at about 37 hz. The CLTF deviates somewhat from 1/B in the vicinity of .01hz, where the loop gain dips to 2. Above the gain-crossover frequency, the transfer function tracks the gain curve of the forward elements, and they are what determine the high frequency portion of the system response. The computations show a phase-margin of better than 87 degrees, which indicates that the loop, *as described*, is pleasantly far from oscillation. In reality, the components which have been ignored, those acting at the higher frequencies, are precisely those which will most strongly influence the loop stability. *More analysis to be done here.*

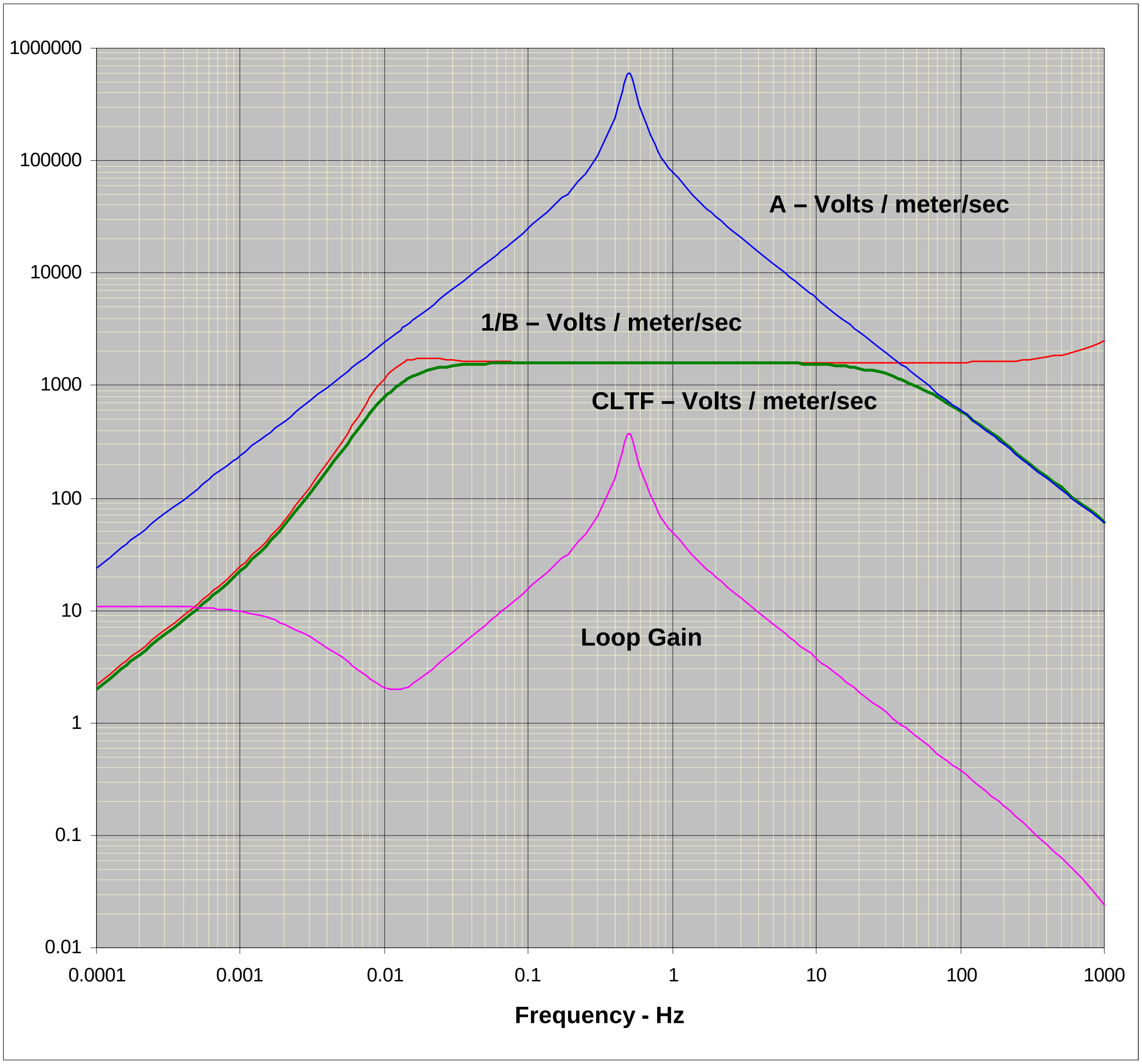

Figure 5 – Loop Functions with respect to velocity – Volts / meter/sec

The input is the velocity of the seismic mass due to ground-motion, and the feedback is the velocity imparted to the seismic mass by the feedback coil.

Evaluating asymptotes:

In the log-log graph of figure 5 the various curves appear to consist of straight line segments, connected by relatively short curved sections. These straight lines are called the asymptotes to the functions, as shown in Figure 6. We note that all these lines have slopes which are integers. Some are horizontal (slope = 0) some have slope ± 1 (x 10 or x 0.1 per frequency decade, i.e. ±20db /decade), and some have slope ± 2 (x 100 or x 0.01, i.e. ±40db per decade). It is not difficult to compute the equations for the asymptotes. Knowing them is particularly useful when one wants to understand what factors are contributing to what characteristics of the system.

For the system as described, the asymptote lines in Figure 6 are:

for the forward function :

for $0.0001 < f < 0.1$ Hz $A \cong 2\pi f\ M r / K$ Volts / m/sec

for $2 < f < 1000$ Hz $A \cong r / 2\pi f$ Volts / m/sec

for the feedback term 1/B:

for $f < 0.001$ Hz $1/B \cong 2\pi f\ M /(G_n(1/R_p + 1/R_I))$ Volts / m/sec

for $0.002 < f < 0.01$ Hz $1/B \cong (2\pi f)^2\ M\ R_I\ \tau_I / G_n$ Volts / m/sec

for $0.04 < f < 10$ Hz $1/B \cong M / (C_d\ G_n)$ Volts / m/sec = *velocity response*

The system response (CLTF) ≅ 1/B below 10 Hz and ≅ A above 70 Hz

The asymptotes to the Loop Gain function may be computed for the appropriate frequency ranges by using Loop Gain ≡ AB = A / (1/B) and applying it to the asymptotes for A and 1/B.

In this log plot it is useful to note that the vertical distance between A and 1/B which = A/(1/B) = AB is the loop gain. The point where A intersects 1/B is where AB = 1, which defines the gain crossover frequency.

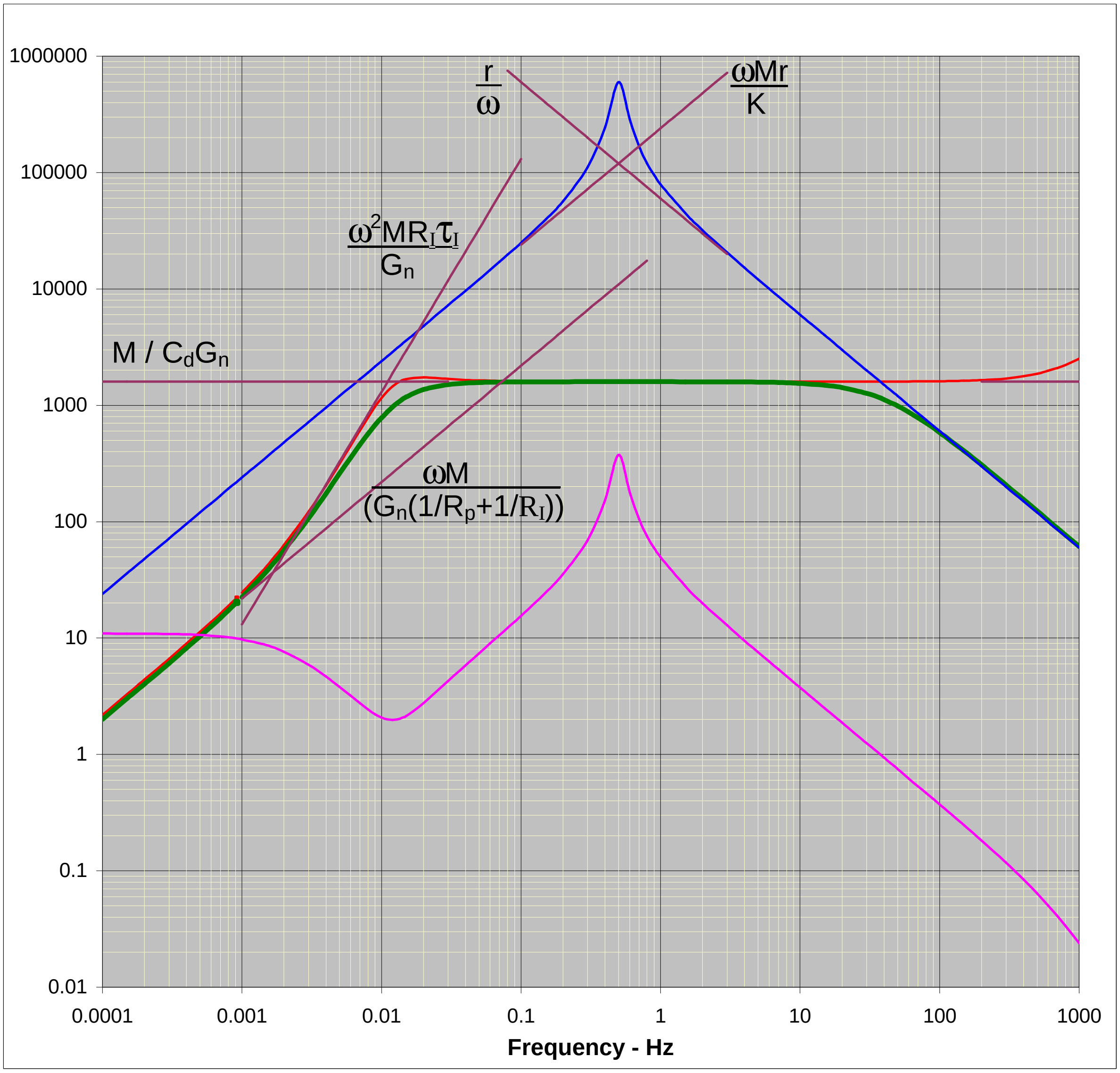

Figure 6 – Loop Functions showing asymptotes

Note that an asymptote slope of +1 is associated with a function which contains ω, and +2 with a function containing ω^2. Slopes of -1 have a $1/\omega$ term and -2 would contain $1/\omega^2$. The horizontal asymptote is a constant and thus has no ω term.

Evaluating Corner Frequencies:

Another useful characteristic of the asymptotes is that they intersect at frequencies which are significant to the loop, i.e. corresponding to poles and zeros of the transfer function. To find the corner frequencies, the equations for the intersecting asymptotes can easily be set equal and solved for frequency.

• For example equating the rising and falling asymptotes of A shows that they intersect at

$f = \frac{\sqrt{K/M}}{2\pi} = 0.500$ Hz Which is the natural frequency we assumed for the spring-mass.

• The upward bend of 1/B occurs at $f = \frac{1}{2\pi} \frac{1}{\tau_I (R_I/R_p + 1)} = 0.0017$ Hz

which is of no special significance as far as I can see.

•The low frequency rolloff of 1/B (recall that 1/B ≅ the system velocity response) is at

$f = \frac{1}{2\pi} \frac{1}{\sqrt{C_d R_I \tau_I}} = 0.011$ Hz which approximates the natural frequency of the system with the loop active (90 second period).

• The high frequency rolloff of the system velocity response occurs where A = 1/B which is at $f = \frac{1}{2\pi} \frac{r C_d G_n}{M} = 37.48$ Hz This, it should also be noted, is the gain crossover frequency.

What's doing what:

From observing the formulas for the asymptote lines and corner frequencies, we can begin to relate individual blocks of the loop to particular portions of the graphs.

The proportional feedback path dominates at the very lowest frequencies. The integrator also contributes proportional feedback below its lower cutoff frequency. This is apparent when we observe that the lowest frequency asymptote to 1/B has a term $(1/R_p + 1/R_I)$.

The integral feedback path dominates between 0.0017 Hz and 0.011 Hz. Its main benefit appears to to be that it pushes down 1/B at the very low frequencies giving higher loop gain in that region. It is also important in that it gives a 40db/decade slope to the velocity response below the low frequency corner. This is the desired “standard” shape for the seismometer velocity response.

The derivative feedback determines the velocity sensitivity in the main response range, 0.011 - 37 Hz. The transition between integral and derivative feedback at 0.011 Hz results in a quadratic term in 1/B and some modest peaking.

Design considerations for VBB vertical seismograph electronics:

1) The aim of the design will be to create a force-feedback system which has a desired velocity response, and which is nearly independent from the characteristics of the mechanical spring-mass system. The phase response of the system (essentially, the time delays for different frequencies) should at least be known, and could possibly be deliberately shaped to match other instruments, if waveform correlation will be important.

2) Since this is a feedback system, the loop must not self-oscillate and should not exhibit any resonant peaking at the gain crossover frequency.

3) We want the system to respond to the largest ground velocities expected, without clipping or other waveform distortion, and without creating false long period signals. Low distortion is particularly important if digital post-processing is planned.

4) Finally, any random signals (noise and drift) added by the system should be smaller than the background noise levels at the planned installation site.

Item (1) relates to the design of the feedback (B) elements. After they have been specified, the loop gain AB should be designed to be as large as reasonably possible over the frequency range where we want the system response and distortion to be tightly controlled.

Item (2) relates to how we get rid of all the loop gain, at the higher frequencies. In particular, the phase lag of the loop gain function at the gain crossover frequency (where AB = 1) must not be too close to 180^o.

Item (3) relates to the clipping levels and nonlinearities of the various loop components. High loop gain helps to linearize the overall loop response. Clipping can occur as a result of either voltage or current limitations.

Item (4) involves choosing the proper electronic components, designing the circuits to minimize the impact of unavoidable component noise and drift, electrical shielding, supply voltage regulation and probably thermal considerations.

Each of these four items will likely involve some compromises with the others, and there's unfortunately still the biggest compromise – cost. For remote systems some additional factors which may need to be considered are size/weight, mechanical ruggedness and power consumption.

Appendix 1 – Magnitude computations

This will outline how the magnitude functions used to plot the graphs may be obtained. In reality I used the COMPLEX, IMABS, IMSUM and related Excel 97 "IM" functions to work directly on the functions of $j\omega$. Remember that everything except s and ω are constants.

Spring-mass

$$\frac{x(s)}{Vel(s)} = \frac{M\,s}{K\,(s^2\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{m / m/s}$$

where τ_0 is the spring-mass time constant - sec/*rad*
T_0 is the undamped natural period in seconds = $2\pi\tau_0$
and ζ is the spring-mass damping factor.

To get the magnitude of the transfer function we set s = $j\omega$, and also indicate magnitudes and replacing j^2 with -1

$$\frac{|x(j\omega)|}{|Vel(j\omega)|} = \frac{M\,|j\omega|}{K\,|(1 - \omega^2\tau_0^2 + j\omega\,2\zeta\tau_0)|} \quad \text{m/ m/s}$$

Evaluating the magnitudes

$$\frac{x(\omega)}{Vel(\omega)} = \frac{M\,\omega}{K\sqrt{((1-\omega^2\tau_0^2)^2 + (2\omega\zeta\tau_0)^2)}} \quad \text{m/ m/s}$$

Total Forward Path

$$A(s) = \frac{V_{out}(s)}{Vel(s)} = \frac{s\,M\,r}{K\,(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{Volts/ m/s}$$

$$|A(j\omega)| = \frac{|j\omega\,M\,r|}{|K\,(j^2\omega^2\tau_0^2 + j\omega\,2\zeta\tau_0 + 1)|} \quad \text{Volts/ m/s}$$

$$A(\omega) = \frac{\omega\,M\,r}{K\sqrt{((1-\omega^2\tau_0^2)^2 + (\omega 2\zeta\tau_0)^2)}} \quad \text{Volts/ m/s}$$

Integrator

$$\frac{I_{out}(s)}{V_{in}(s)} \cong \frac{\omega_I}{R_I\,(s + \omega_I)} \quad \text{Amperes / Volt sec}$$

$$\frac{|I_{out}(j\omega)|}{|V_{in}(j\omega)|} \cong \frac{|\omega_I|}{|R_I(\omega_I + j\omega)|} \qquad \frac{I_{out}(\omega)}{V_{in}(\omega)} \cong \frac{\omega_I}{R_I\sqrt{(\omega_I^2 + \omega^2)}} \qquad \text{Amperes / Volt sec}$$

Derivative Feedback

$$\frac{I_{out}(s)}{V_{in}(s)} = \frac{sC_d}{(sR_cC_d + 1)} \qquad \text{Amperes/ Volt/sec}$$

$$\frac{|I_{out}(j\omega)|}{|V_{in}(j\omega)|} = \frac{|j\omega C_d|}{|(1 + j\omega R_cC_d)|} \qquad \frac{I_{out}(\omega)}{V_{in}(\omega)} = \frac{\omega C_d}{\sqrt{(1 + \omega^2 R_c^2 C_d^2)}} \qquad \text{Amperes/ Volt/sec}$$

If R_c is ignored (its effects are only at high frequencies)

$$\frac{I_{out}(\omega)}{V_{in}(\omega)} \cong \omega C_d \qquad \text{Amperes/ Volt/sec}$$

Total Combined Feedback expression - *Assuming feedback coil resistance is low.*

$$B(s) \cong \frac{G_n}{M} \frac{(R_I + R_p)}{R_I R_p} \frac{1}{s} \frac{\omega_I}{(s + \omega_I)} (s^2 \tau_B^2 + s\, 2\zeta_B\tau_B + 1) \qquad \text{meters/sec / Volt}$$

where $\tau_B^2 = \frac{C_d R_I}{\omega_I} \frac{R_p}{(R_p + R_I)}$ and $\zeta_B = ½\tau_B \left(\omega_I + \frac{1}{R_p C_d}\right)$

$$|B(j\omega)| \cong \frac{G_n}{M} \frac{(R_I + R_p)}{R_I R_p} \frac{1}{|j\omega|} \frac{\omega_I}{|(\omega_I + j\omega)|} |(j^2\omega^2\tau_B^2 + j\omega\, 2\zeta_B\tau_B + 1)| \qquad \text{meters/sec / Volt}$$

$$B(\omega) \cong \frac{G_n}{M} \frac{(R_I + R_p)}{R_I R_p} \frac{\omega_I\sqrt{(1 - \omega^2\tau_B^2)^2 + (\omega\, 2\zeta_B\tau_B)^2}}{\omega\sqrt{(\omega_I^2 + \omega^2)}} \qquad \text{meters/sec / Volt}$$

Mass position filter – 1000 sec hi-pass

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{(s + 1/\tau_P)} = \frac{s}{(s + \omega_p)} \qquad R_p = 1E6, \quad C_p = 1000E\text{-}6$$

where $\tau_P = R_pC_p = 1000$ sec/*rad*. $\omega_p \equiv 1/\tau_p = 0.001$ *rad*/sec

$$\frac{|V_{out}(j\omega)|}{|V_{in}(j\omega)|} = \frac{|j\omega|}{|\omega_p + j\omega|} \qquad \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\omega}{\sqrt{\omega^2 + \omega_p^2}}$$